Weak Keys of the Full MISTY1 Block Cipher for Related-Key Cryptanalysis

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Outline:

- Block Cipher Cryptanalysis
- 2 The MISTY1 Block Cipher
- 3 2103.57 Weak Keys for a Related-Key Differential Attack
- 9 292 Weak Keys for a Related-Key Amplified Boomerang Attack
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1. Block Cipher Cryptanalysis

2. The MISTY1 Block Cipher 3. 2^{103.57} Weak Keys for a Related-Key Differential Attack 4. 2⁹² Weak Keys for a Related-Key Amplified Boomerang Attack 5. Conclusions

1.1 Block Cipher

1.1 Block Cipher

- 1.2 A Cryptanalytic Attack
- 1.3 Four Cryptanalytic Scenarios
- 1.4 Three Elementary Cryptanalysis Techniques
- 1.5 Advanced Cryptanalysis Techniques

- An important primitive in symmetric-key cryptography.
 - * Main purpose: provide confidentiality A most fundamental security goal.
- An algorithm that transforms a fixed-length data block into another data block of the same length under a secret user key.
 - * Input: plaintext.
 - * Output: ciphertext.
 - * Three sub-algorithms: encryption, decryption, key schedule.
- Constructed by repeating a simple function many times, known as the iterated method.
 - * An iteration: a round.
 - * The repeated function: the round function.
 - * The key used in a round: a round subkey.
 - * The number of iterations: the number of rounds.
 - * The round subkeys are generated from the user key under a key schedule algorithm.

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1.2 A Cryptanalytic Attack

- An algorithm that distinguishes a cryptosystem from a random function.
- Usually measured using the following three metrics:
 - * Data complexity
 - The numbers of plaintexts and/or ciphertexts required.
 - * Memory (storage) complexity
 - The amount of memory required.
 - * Time (computational) complexity
 - The amount of computation or time required, how many encryptions/decryptions or memory accesses.
- Goals:
 - * Break a cryptosystem (ideally, in a practical complexity).
 - * Enable more secure cryptosystems to be designed.

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1.3 Four Cryptanalysis Scenarios

Ciphertext-only attack scenario

- * Have access to a number of ciphertexts.
- Known-plaintext attack scenario
 - * Have access to a number of ciphertexts and the corresponding plaintexts.

• Chosen-plaintext/cipertext attack scenario

* Can choose a number of plaintexts (or ciphertexts), and be given the corresponding ciphertexts (or plaintexts).

Adaptive chosen plaintext and ciphertext attack scenario

* Can choose plaintexts (or ciphertexts) and be given the corresponding ciphertexts (or plaintexts). Based on the information obtained, the attacker can then choose further plaintexts/ciphertexts, and be given the corresponding ciphertexts/plaintexts ...

- 1.1 Block Cipher
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1.4 Three Elementary Cryptanalysis Techniques

Assume an *n*-bit block cipher with a *k*-bit user key $E_{\mathcal{K}}(\cdot)$.

• A dictionary attack

- * Build a table of all possible ciphertexts corresponding to one particular plaintext, with one entry for each possible key: $C_i = E_{K_i}(P)$.
- * Data: 2^k ciphertexts, Memory: 2^k *n*-bit, Time: negligible.

• A codebook attack:

- * Build a table of the ciphertexts for all the plaintexts encrypted using one unknown key: $C_i = E_K(P_i)$.
- * Data: 2ⁿ plaintext-ciphertext pairs, Memory: 2ⁿ n-bit, Time: negligible.

• An exhaustive key search (or brute force search) attack:

- * Try every possible key, given a known plaintext-ciphertext pair. The correct key will yield the correct correspondence: $E_{\kappa_i}(P) \stackrel{?}{\to} C$.
- * Data: negligible, Memory: negligible, Time: 2^k encryptions.

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1.5 Advanced Cryptanalysis Techniques

An attack is commonly regarded as effective if it is faster than an exhaustive key search.

A trade-off between data, time and/or memory.

- Meet-in-the-middle attack
 - * Reflection-meet-in-the-middle attack, Higher-order meet-in-the-middle attack
- Differential cryptanalysis
 - * Truncated differential, Higher-order differential, Impossible differential
 - * Boomerang, Amplified boomerang, Rectangle attacks, Impossible boomerang
- Linear cryptanalysis
- Differential-linear cryptanalysis
- Integral cryptanalysis
 - * Square attack, Saturation attack
- Slide attack, Reflection attack
- Related-key attack
- Algebraic cryptanalysis

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1.5.1 Differential Cryptanalysis

- Introduced in 1990 by Biham and Shamir.
- Work in a chosen-plaintext/ciphertext attack scenario.
- Take advantage of how a specific difference in a pair of plaintexts can affect a difference in the pair of ciphertexts (under the same key).
- A differential is the combination of the input difference and the output difference.
- The probability of the differential (α, β) for an *n*-bit block cipher \mathbb{E} , written $\Delta \alpha \rightarrow \Delta \beta$, is

$$\mathsf{Pr}_{\mathbb{E}}(\Delta \alpha \to \Delta \beta) = \Pr_{P \in \{0,1\}^n}(\mathbb{E}(P) \oplus \mathbb{E}(P \oplus \alpha) = \beta).$$

• For a random function, the expected probability of any differential is 2^{-n} .

If $\Pr_{\mathbb{E}}(\Delta \alpha \to \Delta \beta) > 2^{-n}$, we can use the differential to distinguish \mathbb{E} from a random function.

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1.5.2 Related-Key (Differential) Cryptanalysis

- Independently introduced by Knudsen in 1992 and Biham in 1993.
- Different from differential cryptanalysis: The pair of ciphertexts are obtained by encrypting the pair of plaintexts using two different keys with a particular relationship, e.g. certain difference.
- Probability of a related-key differential:

$$\mathsf{Pr}_{\mathbb{E}_{\mathcal{K}},\mathbb{E}_{\mathcal{K}'}}(\Delta\alpha\to\Delta\beta)=\Pr_{P\in\{0,1\}^n}(\mathbb{E}_{\mathcal{K}}(P)\oplus\mathbb{E}_{\mathcal{K}'}(P\oplus\alpha)=\beta).$$

• For a random function, the expected probability of any related-key differential is 2^{-n} .

If $\Pr_{\mathbb{E}_{K},\mathbb{E}_{K'}}(\Delta \alpha \to \Delta \beta) > 2^{-n}$, we can use the related-key differential to distinguish \mathbb{E} from a random function.

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1.5.3 Amplified Boomerang Attack

- Introduced in 2000 by Kelsey, Kohno and Schneier (as a variant of the boomerang attack).
- Work in a chosen-plaintext/ciphertext attack scenario.
- Based on an amplified boomerang distinguisher:
 - * Treat a block cipher \mathbb{E} as a cascade of two sub-ciphers $\mathbb{E} = \mathbb{E}^0 \circ \mathbb{E}^1$.
 - * Defined to be a pair of differentials $(\Delta \alpha \rightarrow \Delta \beta, \Delta \gamma \rightarrow \Delta \delta)$:
 - $\Delta \alpha \rightarrow \Delta \beta$ for \mathbb{E}^0 with probability *p*; $\Delta \gamma \rightarrow \Delta \delta$ for \mathbb{E}^1 with probability *q*.
 - * Concerned event: $\mathbb{E}(P) \oplus \mathbb{E}(P') = \delta$ and $\mathbb{E}(P \oplus \alpha) \oplus \mathbb{E}(P' \oplus \alpha) = \delta$
 - * Probability: $p^2q^22^{-n}$ approximately (under assumptions).
- For a random function, the expected probability of any amplified boomerang distinguisher is 2^{-2n} .

If $p^2q^2 > 2^{-n}$, we can use the distinguisher to distinguish between \mathbb{E} and a random function.

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An Amplified Boomerang Distinguisher



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1.5.4 Related-Key Amplified Boomerang Attack

- A combination of the amplified boomerang attack and related-key cryptanalysis.
- Based on a related-key amplified boomerang distinguisher.
 - * Treat a block cipher \mathbb{E} as $\mathbb{E} = \mathbb{E}^0 \circ \mathbb{E}^1$.
 - * Work typically in a related-key attack scenario with four related keys K_A, K_B, K_C, K_D :
 - $K_A \oplus K_B = K_C \oplus K_D;$
 - $K_A \oplus K_C = K_B \oplus K_D.$
 - * Consist of four related-key differentials.
 - * Concerned event: $\mathbb{E}_{\mathcal{K}_{\mathcal{A}}}(P) \oplus \mathbb{E}_{\mathcal{K}_{\mathcal{C}}}(P') = \delta$ and $\mathbb{E}_{\mathcal{K}_{\mathcal{B}}}(P \oplus \alpha) \oplus \mathbb{E}_{\mathcal{K}_{\mathcal{D}}}(P' \oplus \alpha) = \delta$.
 - * Probability: $p^2q^22^{-n}$ approximately (under assumptions).
- For a random function, the expected probability of any related-key amplified boomerang distinguisher is 2^{-2n} .

If $p^2q^2 > 2^{-n}$, we can use the distinguisher to distinguish between $\mathbb E$ and a random function.

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A Related-Key Amplified Boomerang Distinguisher



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2.1 Introduction

- Designed by Mitsubishi (Matsui et al.), published in 1995.
- A 64-bit block cipher, a user key of 128 bits, and a recommended number of 8 rounds, with a total of 10 key-dependent logical functions **FL**:

2.1 Introduction

2.4 Security

- * two FL functions at the beginning;
- * two FL functions inserted after every two rounds.
- A Japanese CRYPTREC-recommended e-government cipher, an European NESSIE selected cipher, an ISO international standard.
- Widely used in Mitsubishi products as well as in Japanese military.

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2.2 Structure



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2.3 Key Schedule

1. Represent a user key K as eight 16-bit words $K = (K_1, K_2, \dots, K_8)$.

2.2 Structure 2.3 Key Schedule 2.4 Security

2. Generate a different set of eight 16-bit words K_1', K_2', \cdots, K_8' by

$$K'_i = \mathbf{FI}(K_i, K_{i+1}), \text{ for } i = 1, 2, \cdots, 8.$$

3. Subkeys:

$$\begin{split} & KO_{i1} = K_i, KO_{i2} = K_{i+2}, KO_{i3} = K_{i+7}, KO_{i4} = K_{i+4}; \\ & KI_{i1} = K'_{i+5}, KI_{i2} = K'_{i+1}, KI_{i3} = K'_{i+3}; \\ & KL_i = K_{\frac{i+1}{2}} ||K'_{\frac{i+1}{2}+6}, \text{ for } i = 1, 3, 5, 7, 9; \text{otherwise}, KL_i = K'_{\frac{i}{2}+2} ||K_{\frac{i}{2}+4}. \end{split}$$

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2.4 Security	

- Has been extensively analysed against a variety of cryptanalytic methods.
- No whatever cryptanalytic attack on the full version.

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3.1 Related Work

3.1 Related Work

- 3.2 A Corrected Class of Weak Keys and Improved 7-Round Related-Key Diff.
- 3.3 Attacking the Full MISTY1 under Weak Keys 3.4 Another Class of 2^{102.57} Weak Keys

Dai and Chen's related-key differential attack on 8-round MISTY1 with only the last 8 FL functions (INSCRYPT 2011).

- A class of 2¹⁰⁵ weak kevs.
 - * A weak key is a user key under which a cipher is more vulnerable to be attacked.
- A 7-round related-key differential characteristic with probability 2^{-60} .
- Attacking the 8-round reduced version under weak keys.
 - * Attack procedure is straightforward, by conducting a key recovery on FO₁ in a way similar to the early abort technique for impossible differential cryptanalysis.
 - * Data complexity: 263 chosen ciphertexts.
 - * Memory complexity: 2³⁵ bytes.
 - * Time complexity: 2^{86.6} encryptions.

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3.1.1 A Class of 2¹⁰⁵ Weak Keys

Three binary constants:

- * 7-bit a = 0010000:
- * 16-bit b = 001000000010000:

Let K_A , K_B be two 128-bit user keys:

$$K_A = (K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8),$$

$$K_B = (K_1, K_2, K_3, K_4, K_5, K_6^*, K_7, K_8).$$

Let K'_{A}, K'_{B} be the corresponding 128-bit words generated by the key schedule:

$$\begin{split} & \mathcal{K}'_A = (\mathcal{K}'_1, \mathcal{K}'_2, \mathcal{K}'_3, \mathcal{K}'_4, \mathcal{K}'_5, \mathcal{K}'_6, \mathcal{K}'_7, \mathcal{K}'_8), \\ & \mathcal{K}'_B = (\mathcal{K}'_1, \mathcal{K}'_2, \mathcal{K}'_3, \mathcal{K}'_4, \mathcal{K}'^5, \mathcal{K}'^6, \mathcal{K}'_7, \mathcal{K}'_8). \end{split}$$

The class of weak keys is defined to be the set of all possible (K_A, K_B) satisfying the following 10 conditions:

$$\begin{array}{ll} {{\cal K}_6 \oplus {\cal K}_6^* = c,} & {{\cal K}_5' \oplus {\cal K}_5^{**} = b,} & {{\cal K}_6' \oplus {\cal K}_6'^* = c,} & {{\cal K}_{6,12} = 0,} & {{\cal K}_{7,3} = 1,} \\ {{\cal K}_{7,12} = 0,} & {{\cal K}_{8,3} = 1,} & {{\cal K}_{4,3}' = 1,} & {{\cal K}_{4,12}' = 1,} & {{\cal K}_{7,3}' = 0.} \end{array}$$

The number:

$$|{\it K}_1|=2^{16}, |{\it K}_2|=2^{16}, |{\it K}_3|=2^{16}, |({\it K}_4,{\it K}_5)|=2^{30}, |({\it K}_6,{\it K}_7,{\it K}_8)|=2^{27}.$$

Therefore, a total of 2¹⁰⁵ weak keys.

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3.1.2 A 7-Round Related-Key Differential Characteristic



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3.2 A Corrected Class of Weak Keys

Focus on the 7-round related-key differential characteristic.



Not all the 2¹⁵ possible K'_7 (i.e. KI_{21}) defined by the weak key class make $\Pr_{\mathbf{FI}_{21}}(\Delta b \to \Delta c) > 0!$ The number of K'_7 defined by the weak key class is 2^{15} , the number of K'_7 satisfying $\Pr_{\mathbf{FI}_{21}}(\Delta b \to \Delta c) > 0$ is about $2^{14.57}$. The number of K'_7 defined by the weak key class & satisfying $\Pr_{\mathbf{FI}_{21}}(\Delta b \to \Delta c) > 0$ is about $2^{13.57}$. $\Pr_{\mathbf{FI}_{21}}(\Delta b \to \Delta c) = 2^{-15}/2^{-14}/2^{-13.42}.$

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3.1 Related Work

- 3.2 A Corrected Class of Weak Keys and Improved 7-Round Related-Key Diff.
- 3.3 Attacking the Full MISTY1 under Weak Keys
- 3.4 Another Class of 2^{102.57} Weak Keys



Not all the 2¹⁶ possible K'_2 (i.e. KI_{73}) defined by the weak key class make $\Pr_{\mathbf{FI}_{73}}(\Delta c \to \Delta c) > 0$! The number of K'_2 defined by the weak key class is 2¹⁶, the number of K'_2 satisfying $\Pr_{\mathbf{FI}_{21}}(\Delta b \to \Delta c) > 0$ is 2¹⁵. The number of K'_2 defined by the weak key class & satisfying $\Pr_{\mathbf{FI}_{73}}(\Delta c \to \Delta c) > 0$ is 2¹⁵. $\Pr_{\mathbf{FI}_{73}}(\Delta c \to \Delta c) = 2^{-15}$.

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As a result,

• A class of
$$2^{102.57}$$
 weak keys:
 $|K_1| = 2^{16}, |(K_2, K_3)| = 2^{31}, |(K_4, K_5)| = 2^{30}, |(K_6, K_7, K_8)| \approx 2^{25.57}$
* $|K_3| = 2^{16}, |K_5| = 2^{16}$
* $|K_7'| = 2^{13.57}; \forall K_7', \exists 2^{12} (K_6', K_8)$.
* $|K_{2,8-16}'| = 2^8, |K_3'| = 2^{16}, |K_{4,8-16}'| = 2^8$.

• A 7-round related-key differential with probability 2^{-58} .

*
$$(b||0^{32}||c) \rightarrow (0^{32}||c||0^{16})$$

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3.1 Related Work

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3.3.1 Precomputation

Hash table \mathcal{T}_1 :



Memory complexity: $2^{75.91}$ bytes; Time complexity: $2^{73.59}$ **FI** computations. For every (x, η, X) , there are 2^{23} satisfying $(K_1, K_3, K'_{2,8-16})$ on average.

Hash table \mathcal{T}_2 :

Y: output difference of \mathbf{FI}_{13}

Store satisfying (K_6, K_7, K_8) into Table \mathcal{T}_2 indexed by $(x, \eta, Y, K_1, K'_{4,8-16})$

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Memory complexity: 2^{84.74} bytes; Time complexity: 2^{84.16} FI computations. For every $(x, \eta, Y, K_1, K'_{4,8-16})$, there are $2^{9.57}$ satisfying (K_6, K_7, K_8) on average.

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3.3.2 Attack Outline



3.3.3 Attack Complexity

3.1 Related Work

- 3.2 A Corrected Class of Weak Keys and Improved 7-Round Related-Key Diff.
- **3.3 Attacking the Full MISTY1 under Weak Keys** 3.4 Another Class of 2^{102.57} Weak Keys

• Data complexity: 2⁶¹ chosen ciphertexts.

- Memory complexity: 2^{99.2} bytes.
- Time complexity: 2^{87.94} encryptions.
- Success probability: 76%.

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3.4 Another Class of 2^{102.57} Weak Keys

Focus on the 7-round related-key differential characteristic:



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4.1 Related Work

4.1 Related Work

- 4.2 An Improved 7-Round Distinguisher
- 4.3 Attacking the Full MISTY1 under Weak Keys 4.4 Three Other Classes of 2⁹⁰ Weak Keys

Chen and Dai's related-key amplified boomerang attack on 8-round MISTY1 with only the first 8 FL functions (CHINACRYPT 2011).

- A class of 2⁹⁰ weak keys.
- A 7-round related-key amplified boomerang distinguisher with probability 2^{-118} .
- Attacking the 8-round reduced version under weak keys.
 - * Attack procedure is straightforward, by conducting a key recovery on FO₈ in a way similar to the early abort technique.
 - * Data complexity: 263 chosen plaintexts.
 - * Memory complexity: 2⁶⁵ bytes.
 - * Time complexity: 2⁷⁰ encryptions.

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4.1 Related Work

- 4.2 An Improved 7-Round Distinguisher
- 4.3 Attacking the Full MISTY1 under Weak Keys 4.4 Three Other Classes of 2⁹⁰ Weak Keys

4.1.1 A Class of 2⁹⁰ Weak Keys

Let K_A , K_B , K_C , K_D be four 128-bit user keys: $K_A = (K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8), \quad K_B = (K_1, K_2^*, K_3, K_4, K_5, K_6, K_7, K_8),$ $K_{C} = (K_{1}, K_{2}, K_{3}, K_{4}, K_{5}, K_{5}^{*}, K_{7}, K_{8}), K_{D} = (K_{1}, K_{7}^{*}, K_{3}, K_{4}, K_{5}, K_{5}^{*}, K_{7}, K_{8}).$ Let K'_A, K'_B, K'_C, K'_D be the corresponding 128-bit words generated by the key schedule:

$$\begin{array}{ll} K_A' = (K_1', K_2', K_3', K_4', K_5', K_6', K_7', K_8'), & K_B' = (K_1'^*, K_2'^*, K_3', K_4', K_5', K_6', K_7', K_8'), \\ K_C' = (K_1', K_2', K_3', K_4', K_5'^*, K_6'^*, K_7', K_8'), & K_D' = (K_1^{+*}, K_2^{+*}, K_3', K_4', K_5^{+*}, K_6^{+*}, K_7', K_8'). \end{array}$$

The class of weak keys is defined to be the set of all possible (K_A, K_B, K_C, K_D) satisfying the following 12 conditions:

$$\begin{array}{ll} K_2\oplus K_2^*=c, & K_6\oplus K_6^*=c, & K_1'\oplus K_1'^*=b, & K_5'\oplus K_5'^*=b\\ K_2'\oplus K_2'^*=c, & K_6'\oplus K_6'^*=c, & K_{5,3}=1, & K_{5,12}=0, \\ K_{4,3}'=0, & K_{7,3}=1, & K_{7,12}=0, & K_{8,3}=0. \end{array}$$

The number:

$$|{\cal K}_1|=2^{16}, |({\cal K}_2,{\cal K}_3)|=2^{16}, |({\cal K}_4,{\cal K}_5)|=2^{29}, |({\cal K}_6,{\cal K}_7)|=2^{14}, |{\cal K}_8|=2^{15}.$$

Therefore, a total of 290 weak keys.

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4.1.2 A 7-Round Related-Key Amp. Boo. Distinguisher

A 7-round related-key amplified boomerang distinguisher with probability $p^2 q^2 2^{-n} = 1^2 \times (2^{-27})^2 \times 2^{-64} = 2^{-118}$ under weak keys.

- \mathbb{E}_0 : Rounds 1 –2, including FL₄ but excluding FL₃.
- * E₁: Rounds 3 −7, including FL₃ (but excluding FL₄).
- Related-key differential $\Delta \alpha \rightarrow \Delta \beta$ for \mathbb{E}_0 : $(0^{48}||b) \rightarrow (0^{32}||c||0^{16})$ with probability 1.
- Related-key differential $\Delta \gamma \rightarrow \Delta \delta$ for \mathbb{E}_1 : $(0^{48}||b) \rightarrow 0$ with probability 2^{-27} . *

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The Two Related-Key Differentials Used



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4.2 An Improved 7-Round Distinguisher

Focus on the second related-key differential:



Surprisingly, all the possible (K'_2, K'^*_2) (i.e. KI_{73}) defined by the weak key class make $\Pr_{\mathbf{FI}_{72}}(\Delta c \to \Delta c) > 0!$ $\Pr_{\mathbf{FI}_{72}}(\Delta c \to \Delta c) = 2^{-15}.$

Thus, a 7-round related-key amplified boomerang distinguisher with probability 2^{-116} . ・ロト ・周ト ・ヨト ・ヨト

4.1 Related Work 4.2 An Improved 7-Round Distinguisher 4.3 Attacking the Full MISTY1 under Weak Keys 4.4 Three Other Classes of 2⁹⁰ Weak Keys

4.3.1 Precomputation

Hash table \mathcal{T}_1 :

 $x \in \{0, 1\}^{32}$: Input of \mathbf{FO}_8 without K_8 .

X: The right 9 bits of the output difference of \mathbf{FL}_{81}

Y: Output difference of \mathbf{FL}_{83}

Store satisfying x into Table \mathcal{T}_1 indexed by (K'_3, K'_5, K_7, X, Y) .



Memory complexity: 2^{79} bytes; Time complexity: 2^{71} FI computations. For every (K'_3, K'_5, K_7, X, Y) , there are 2^8 satisfying x on average.

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Hash table \mathcal{T}_2 :

 $x \in \{0, 1\}^{32}$: Input of \mathbf{FL}_{10}^{-1} . λ : Output of \mathbf{FL}_{10}^{-1} after being xored with $(K_8||0^{16})$.

Store (K_1, K_8) into Table \mathcal{T}_2 indexed first by K_7 and then by (x, λ) .



- 4.1 Related Work
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4.3.2 Attack Outline



- Step 1: Choose two sets of $2^{58.5}$ plaintext pairs with difference $(0^{48}||b)$.
- Step 2: Keep the quartets such that each ciphertext pair has difference (2||0).
- Step 3: Focus on \mathbf{FL}_9 . Guess K'_3 , keep the quartets such that each pair has 7-bit difference a.
- Step 4: Focus on **FL**₉. Guess K_5 , compute (X, Y) and (X^*, Y^*) .
- Step 5: Guess K_7 , get the two possible values for K_6 , and compute K'_5 .
- Step 6: Focus on \mathbf{FI}_{81} and \mathbf{FI}_{83} . Obtain possible inputs to \mathbf{FO}_8 excluding XOR with K_8 from Table \mathcal{T}_1 .
- Step 7: Focus on **FL**₁₀. Obtain (K_1, K_8) from Table \mathcal{T}_2 .
- Step 8: For a subkey guess whose counter is non-zero, exhaustively search the remaining key bits.

4.1 Related Work

4.2 An Improved 7-Round Distinguisher

4.3 Attacking the Full MISTY1 under Weak Keys 4.4 Three Other Classes of 2⁹⁰ Weak Keys

4.3.3 Attack Complexity

- Data complexity: 2^{60.5} chosen plaintexts.
- Memory complexity: 2^{80.07} bytes.
 - * On-line: 278.23;
 - * Off-line: 2^{79.58}
- Time complexity: 2^{80.18} encryptions.
- Success probability: 86%.

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4.4 Three Other Classes of 2⁹⁰ Weak Keys

Focus on the first related-key differential:

Consider the three other possible combinations of $(K_{5,3}, K_{5,12})$, further classified by $(K'_{3,3}, K'_{3,12})$



Thus, a total of 292 weak keys.

5. Conclusions

Have presented related-key differential and amplified boomerang attacks on the full MISTY1 algorithm under certain weak key assumptions.

- * Have described 2^{103.57} weak keys for a related-key differential attack on the full MISTY1.
- Have described 2⁹² weak keys for a related-key amplified boomerang attack on the full MISTY1.
- * Quite theoretical, for the attacks work under the assumptions of weak-key and related-key scenarios and their complexities are very high.

The MISTY1 cipher does not behave like a random function (in the related-key model), and cannot be regarded to be an ideal cipher.

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Summary of Main Cryptanalytic Results

#Rounds	FL	#Keys	Attack Type	Data	Time	Year
6 (1 - 6)	yes	2 ¹²⁸	Impossible differential	2 ⁵¹ CP	2 ^{123.4} Enc.	2008
6 (1 - 6)	yes	2 ¹²⁸	Higher-order differential	$2^{53.7}CP$	2 ^{64.4} Enc.	2008
6 (3 - 8)	yes	2 ¹²⁸	Integral	2 ³² CC	$2^{126.1} Enc.$	2009
7(1-7)	yes	2 ¹²⁸	Higher-order differential	$2^{54.1}CP$	$2^{120.7}$ Enc.	2008
7^{\dagger} (2 - 8)	yes	2 ⁷³	Related-key amplified boomerang	2 ⁵⁴ CP	2 ^{55.3} Enc.	2008
$8^{\dagger} (1-8)$	yes	2 ⁹⁰	Related-key amplified boomerang	2 ⁶³ CP	2 ⁷⁰ Enc.	2011
$8^{\dagger} (1-8)$	yes	2 ^{105‡}	Related-key differential	2 ⁶³ CC	2 ^{86.6} Enc.	2011
full	yes	2 ^{103.57}	Related-key differential	2 ⁶¹ CC	2 ^{87.94} Enc.	2012
		2 ⁹²	Related-key amplified boomerang	2 ^{60.5} CP	2 ^{80.18} Enc.	2012

CP: Chosen Plaintexts, CC: Chosen Ciphertexts, Enc.: Encryptions,

†: Exclude the first/last layer of two FL functions, ‡: There is a flaw.

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Thank you!

Questions or Comments?

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